TEACH YOUR STUDENTS TO REASON ABSTRACTLY AND QUANTITATIVELY

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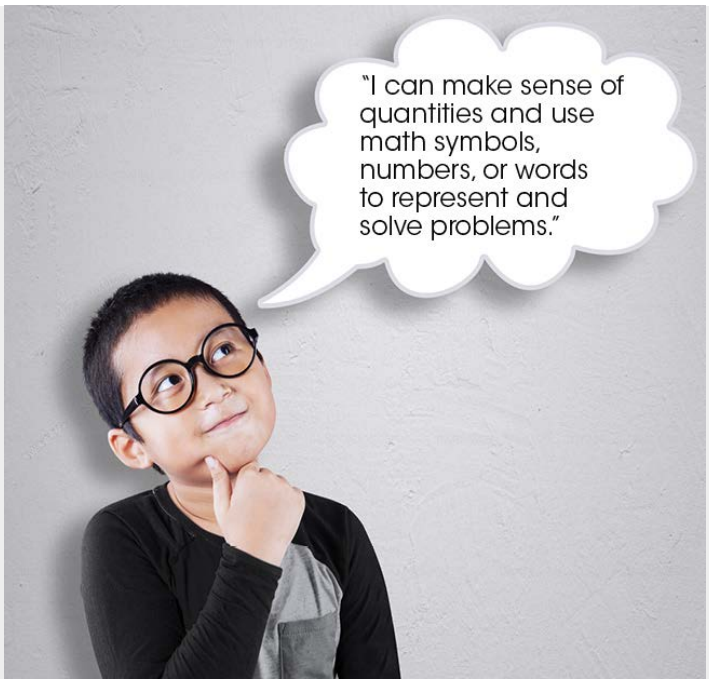

In this article you will ...

- What the metacognitive thinking looks like at each grade level for Mathematical Practice Standard 2.
- A step-by-step approach to teaching Mathematical Practice Standard 2.
- A reflection guide to support students as they "think about their thinking" around Mathematical Practice Standard 2.

Categories: feature, Teaching

Tags: Math Practice 2, math strategies, mathematics, metacognition, Standards for Mathematical Practice, student ownership

Reading Time: 6 minutes

In today's world students who are truly successful in mathematics can not only solve problems, but can explain:

- *what each problem is asking,*
- *how they will solve the problem,*
- *why their solution plan makes sense, and*
- *how they know their answer is correct.*

In other words, students must be able to think metacognitively about each and every problem and explain their thinking.

The Standards for Mathematical Practice present a helpful framework for this complex and important work. That's why we've provided you with a metacognitive study of each of the mathematical practices—what this thinking looks like at each grade level, a step-by-step approach to teaching each practice, and a reflection guide to support students as they "think about their thinking."

This article is focused on...

STANDARD FOR MATHEMATICAL PRACTICE 2: Reason abstractly and quantitatively.

What This Thinking Looks Like at Each Grade Level

Because the 8 Standards for Mathematical Practice are the same for all grade levels, they are not specific enough to provide much practical guidance for classroom implementation. So, we've provided the standard and its learning progression for every grade from Pre-K through High School to better understand how the thinking builds in tandem with students' cognitive development and the demands of mathematical content, as shown in the example below.

elevated achievement GROUP

Practice 2

Reason abstractly and quantitatively.

Learning Progression

PRE-K	Younger students begin to recognize that a number represents a specific quantity. Then, they are supported to connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities.
KINDERGARTEN	Younger students begin to recognize that a number represents a specific quantity. Then, they connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities.
FIRST GRADE	Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities.
SECOND GRADE	Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. Second graders begin to know and use different properties of operations and relate addition and subtraction to length.
THIRD GRADE	Third graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.
FOURTH GRADE	Fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.
FIFTH GRADE	Fifth graders should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.
SIXTH GRADE	In grade 6, students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
SEVENTH GRADE	In grade 7, students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
EIGHTH GRADE	In grade 8, students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
HIGH SCHOOL	High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

A Step-By-Step Approach to Teaching Practice 2

Fostering metacognition requires a balance of explicit instruction, teacher modeling, student-centered exploration, and responsive coaching that helps students first learn the kinds of questions and thought processes they can apply, and then grow to use them on their own. These metacognitive skills come naturally to some students but not to others. Teachers must play an active role in teaching them and helping students own their mathematical learning.

Use these two processes to teach, model, and guide students in mastering Practice 2.

Process to reason abstractly

1. Clarify the context of the problem.
2. Explain how each number or symbol relates to the problem.
3. Translate the situation in the problem into an equation.
4. Sequence order of operations to solve the problem.

Process to reason quantitatively

1. Clarify context of the problem.
2. Identify quantities within the problem.
3. Sequence order of operations.
4. Use quantities and sequence to solve the problem.

A Reflection Guide to Support Students as They "Think About Their Thinking"

As we said before, mathematics is much more than just solving the problems and checking to see if the answer is correct. It also involves each student's understanding of what they are learning, how they can learn it, and what they have learned about their mathematical thinking along the way.

When you're focusing on Practice 2, teach your students this routine and then encourage them to use it for each and every math problem they solve. In doing so, you will support them in thinking about, reflecting on, and sharing their thoughts as the develop ownership of their mathematics.

Practice 2

Reason abstractly and quantitatively.

"I can make sense of quantities and use math symbols, numbers, or words to represent and solve problems."

Reflection

To what degree can you make sense of quantities and use math symbols, numbers, or words to represent and solve problems?

1	2	3	4	5
never		sometimes		always

- What does "reason abstractly" mean?
- What does "reason quantitatively" mean?
- How do you make sense of quantities?
- How do you use math symbols, numbers, or words to represent and solve problems?
- How does reasoning abstractly and quantitatively help you?

Foster Learnership in Mathematics

Owning their mathematics won't be automatic. On the contrary—most students need a great deal of explicit instruction, modeling, and coaching before they develop the metacognitive habits that allow them to take ownership of their learning. The more students are able to understand their own learning process and articulate it, the easier it will be for them to improve their conceptual understanding, procedural skill and fluency, and applications. In other words, students will own their mathematics with metacognition and develop learnership.

Click here to download a resource with the processes and reflection for the Standard for Mathematical Practice 2: Reason abstractly and quantitatively. Click below to access the same great resources for the other Standards for Mathematical Practices.

- Make Sense of Problems and Persevere in Solving Them
- Construct Viable Arguments and Critique the Reasoning of Others
- Model with Mathematics
- Use Appropriate Tools Strategically
- Attend to Precision
- Look for and Make Use of Structure
- Look for and Express Regularity in Repeated Reasoning

We hope you check out our resources...use them...share them...continue your learnership.

Continue the Learning

Check out these articles and resources to continue your learning about this topic...

- Our Article, "Building Math Metacognition in Three Easy Steps"
- Our Tools for Teaching the Practices
- Our Posters for Teaching the Mathematical Practices

The Learning Brief

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