Elevated Achievement Group presents…

Solving the Mathematical Practices Puzzle: How to Develop Students' Skills and Metacognition in Math

May 1, 2021

ownership = elevated achievement = increased

Presenters

Elevated Achievement Group

Marilyn M. Saucedo, Ed.D.

msaucedo@elevatedachievement.com

951-536-3758

Lesley Fields

lfields@elevatedachievment.com

817-875-8902

Session Agenda and Outcome

Agenda: This session is designed to answer the following questions:

- ❑ What are the Standards for Mathematical Practice and how do they differ from and work with the Standards for Mathematical Content?
- ❑ Why is integrating these standards critical for students' mathematical development?
- ❑ What does this look like in the classroom?

Outcome: As a result, participants will gain an understanding of tools needed to be able to implement instructional strategies that integrate mathematical practices and content in grades PreK–2.

The Math Wars

"The new standards grew out of a long and heated debate about mathematics learning…known as the 'math wars,' pitting conceptual understanding and sense-making against procedures, rules, and memorization. The new math standards grew out of the decades-long attempts to acknowledge that both were important aspects of the math curriculum. By incorporating both…, they brought together both sides of the math wars, building on 'the best of previous state standards plus a large body of evidence from international comparisons and domestic reports and recommendations to define a sturdy staircase to college and career readiness.' (National Governors Association, 2013)"

— The Practices in Action, 2021

Two Types of Standards

STANDARDS FOR MATHEMATICAL CONTENT	STANDARDS FOR MATHEMATICAL PRACTICE
A list of things students should understand and be able to do by the end of each grade	A list of ways that proficient students engage with mathematics, including thinking skills and habits of mind
• Specific mathematical knowledge and skills that follow a step-by-step learning progression across grade levels and courses	• More general processes and proficiencies that evolve over time, influenced by cognitive development and the sophistication of the content
• K–8 organized by grade level; high school organized by conceptual theme	• Standards are the same across all grade levels
• Familiar to most teachers	• Not as familiar to teachers
• Easily and frequently test, and therefore the focus of the typical math curriculum	• Not as easily or frequently tested, and therefore often neglected in the math curriculum

Defining the Practices for Metacognition

Students are supported by lessons that let them own the phases of learning.

NCTM Process Standards

Problem Solving—developing a variety of strategies to solve problems that arise in mathematics and in other contexts

Reasoning and Proof—using various types of reasoning to investigate, develop, and evaluate mathematical arguments and proofs

Communication—using the language of mathematics to communicate thinking and to evaluate the thinking of others

Connections—understanding how mathematical ideas interconnect and apply to other contexts

Representation—using mathematical representations to solve problems and to model, interpret, and communicate ideas

(National Council of Teachers of Mathematics, 2000)

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

(National Governor's Association, 2010)

National Research Council Strands of Proficiency

Conceptual Understanding—comprehension of mathematical concepts, operations, and relations

Procedural Fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

Strategic Competence—ability to formulate, represent, and solve mathematical problems

Adaptive Reasoning—capacity for logical thought, reflection, explanation, and justification

Representation—using mathematical representations to solve problems and to model, interpret, and communicate ideas

(National Research Council, 2001)

Building Math Muscle Memory

"One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics and may have a better chance to succeed at a less familiar task such as expanding $(a + b)(x + y)$."

— National Governor's Association, 2013

Learning Progressions

STANDARD FOR MATHEMATICAL PRACTICE 1:

Make sense of problems and persevere in solving them.

I can determine what the problem is asking me to do and not give up until I've solved it.

Practice 1

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

In short, mathematically proficient students:

- Interpret and make meaning of the problem to find a starting point.
- Analyze what is given in order to explain to themselves the meaning of the problem.
- Plan a solution pathway instead of jumping to a solution.
- Monitor their own progress and change the approach if necessary.
- See relationships between various representations.
- Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another.
- Continually ask themselves, "Does this make sense?"
- Can understand various approaches to solutions.

National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010c

Learning Progressions

Practice 1 Make sense of problems and persevere in solving them.

Learning Progression

KINDERGARTEN	In Kindergarten, students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" or they may try another strategy.
FIRST GRADE	In first grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They are willing to try other approaches.
SECOND GRADE	In second grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. They may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They make conjectures about the solution and plan out a problem-solving approach.
THIRD GRADE	In third grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
FOURTH GRADE	In fourth grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.

FIFTH GRADE	In fifth grade, students solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
SIXTH GRADE	In grade 6, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
SEVENTH GRADE	In grade 7, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
EIGHTH GRADE	In grade 8, students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
HIGH SCHOOL	High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Learning Progressions

Practice 1

Make sense of problems and persevere in solving them.

Learning Progression

KINDERGARTEN	In Kindergarten, students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" or they may try another strategy.
FIRST GRADE	In first grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They are willing to try other approaches.
SECOND GRADE	In second grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. They may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They make conjectures about the solution and plan out a problem-solving approach.

FIFTH GRADE	In fifth grade, students solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"

Questions to Foster Metacognition

Practice 1

Make sense of problems and persevere in solving them.

First Grade

The Practice in Action: When presented with a problem involving adding and subtracting within 20, first graders utilizing this practice explain what the problem is asking them to do and look for ways to solve it. If they get stuck, they use concrete objects, such as cubes and counters to conceptualize and solve the problem. As a way of checking their thinking, they ask themselves "Does this make sense?"

What is 12 – 5?

TEACHER: What is the problem asking you to do?

STUDENT: *The problem is asking me to find the answer to twelve minus five.*

TEACHER: How do you want to solve it?

STUDENT: *I can draw a group of twelve circles, X out five, and count how many are left.*

TEACHER: What can you do if you get stuck?

STUDENT: *If I get stuck, I can use counters instead of my drawing. The answer is seven, but I can check my work by adding five plus seven and make sure it equals twelve. Then I know I have the answer.*

TEACHER: How did making sense of the problem and persevering in solving it help you?

STUDENT: *I have to think about what the problem is, first. Then I can try different ways to solve it. I know my answer is right because twelve minus five is the same as taking five things away from a group of twelve and that's what I did.*

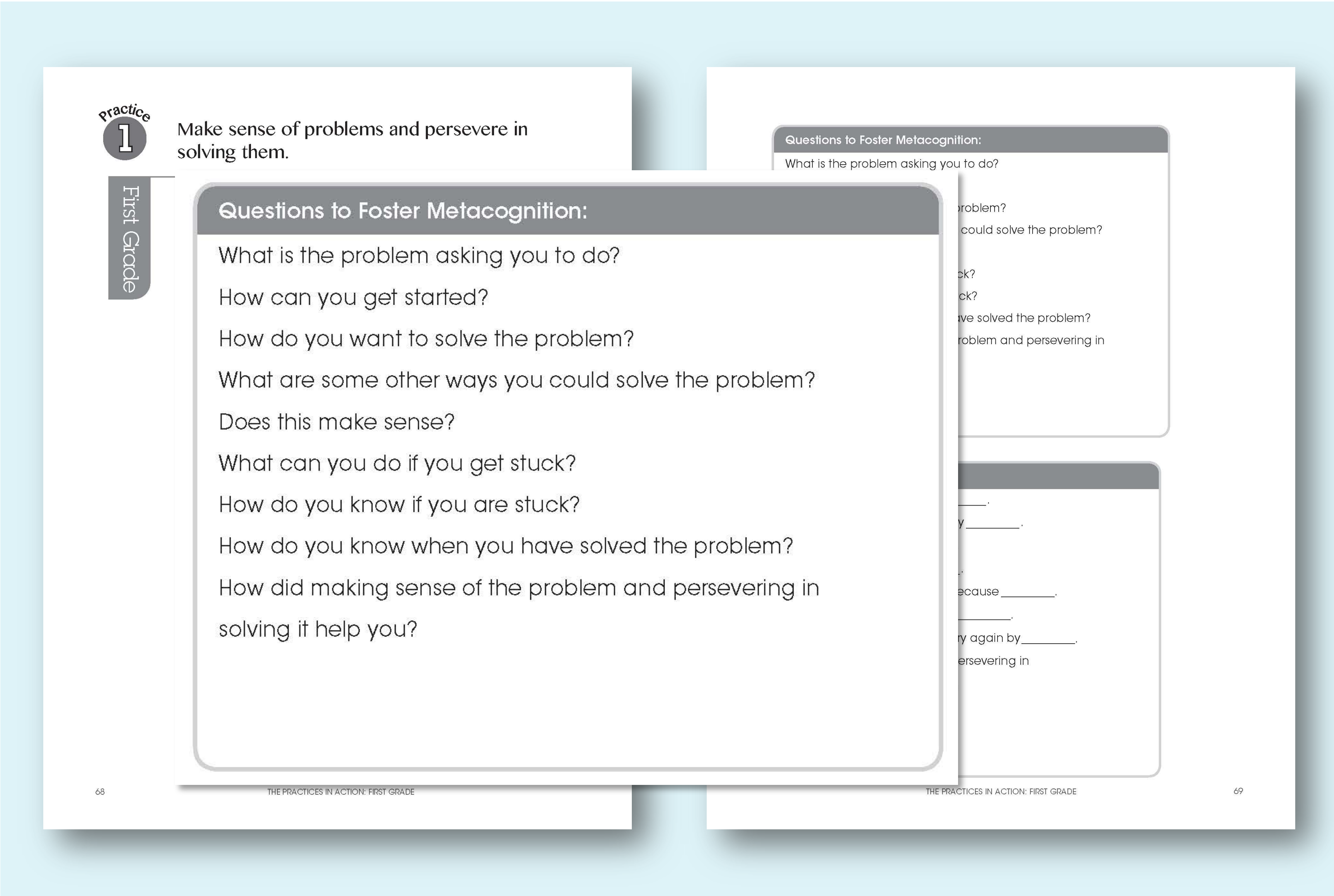
Questions to Foster Metacognition:

What is the problem asking you to do?
How can you get started?
How do you want to solve the problem?
What are some other ways you could solve the problem?
Does this make sense?
What can you do if you get stuck?
How do you know if you are stuck?
How do you know when you have solved the problem?
How did making sense of the problem and persevering in solving it help you?

Ownership Statements:

The problem is asking me to ________.
I can start to solve the problem by ________.
When I am stuck, I can________.
I solved the problem by ________.
I know when I have the answer because ________.
Not giving up helps me because________.
If I can't solve the problem yet, I try again by________.
Making sense of problems and persevering in solving them helps me ________.

Questions to Foster Metacognition



Practice 1

Make sense of problems and persevere in solving them.

First Grade

Questions to Foster Metacognition:

What is the problem asking you to do?

How can you get started?

How do you want to solve the problem?

What are some other ways you could solve the problem?

Does this make sense?

What can you do if you get stuck?

How do you know if you are stuck?

How do you know when you have solved the problem?

How did making sense of the problem and persevering in solving it help you?

68 THE PRACTICES IN ACTION: FIRST GRADE

Questions to Foster Metacognition:

What is the problem asking you to do?

THE PRACTICES IN ACTION: FIRST GRADE 69

Ownership Statements

Practice 1

Make sense of problems and persevere in solving them.

First Grade

Ownership Statements:

The problem is asking me to _________.

I can start to solve the problem by _________.

When I am stuck, I can _________.

I solved the problem by _________.

I know when I have the answer because _________.

Not giving up helps me because _________.

If I can't solve the problem yet, I try again by _________.

Making sense of problems and persevering in

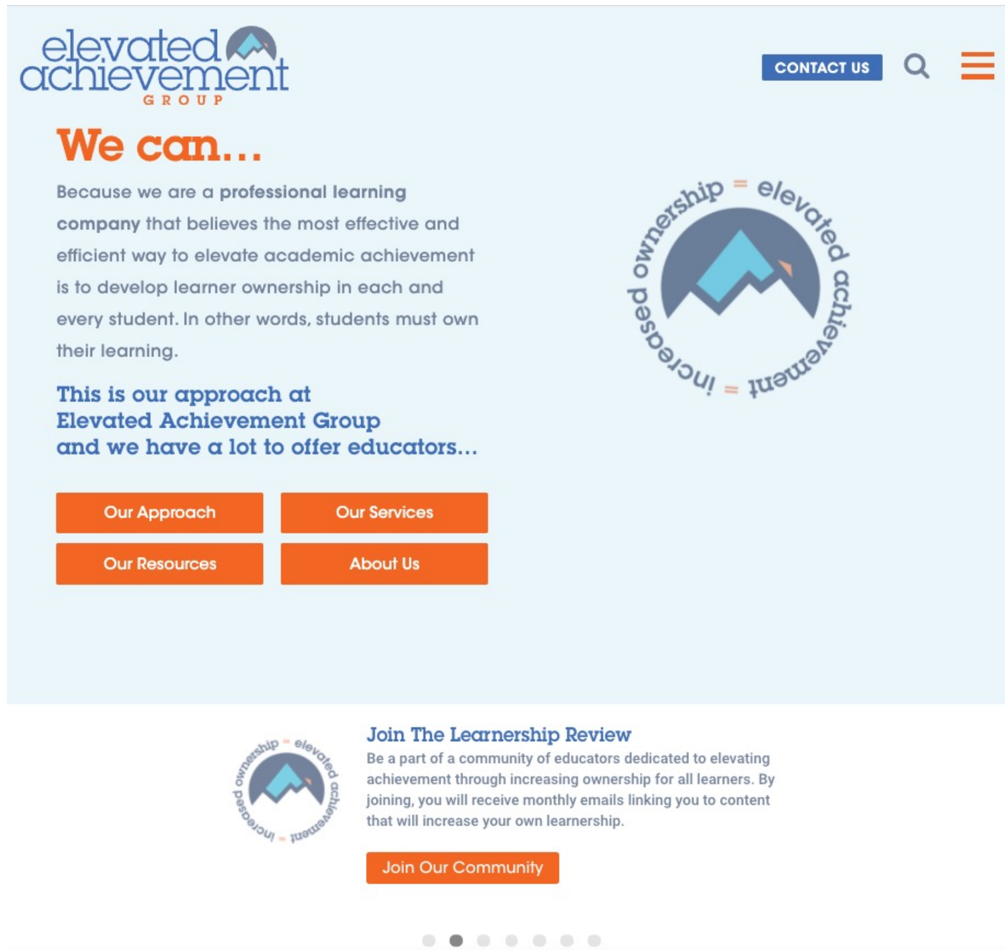
solving them helps me _________.

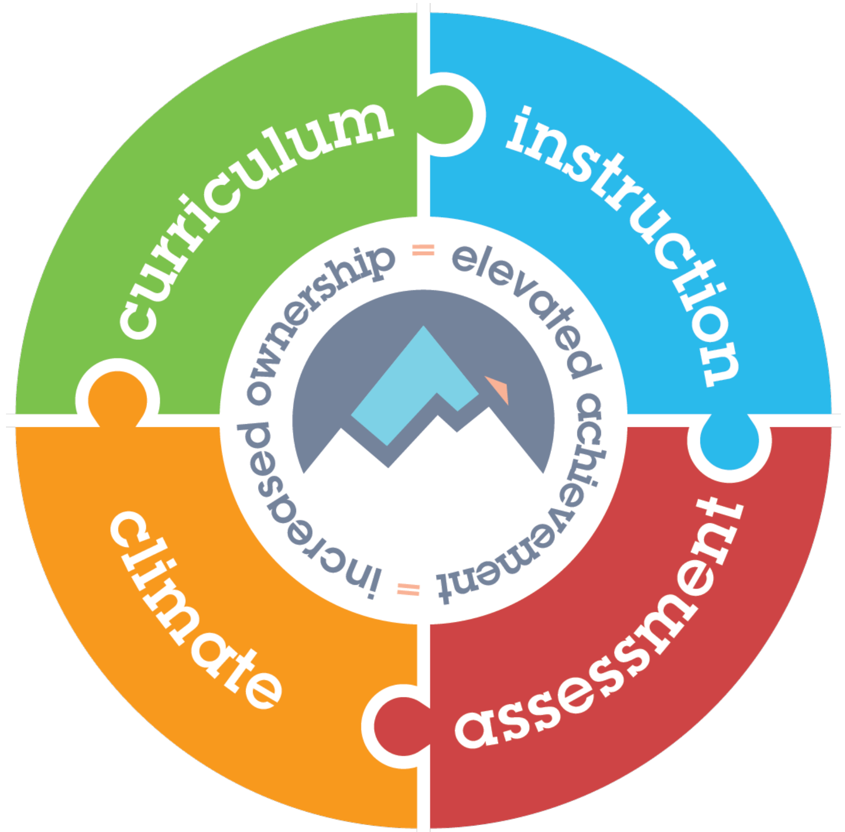
Questions to Foster Metacognition:

What is the problem asking you to do?

Download this presentation and other resources at:

www.elevatedachievement.com





elevated achievement GROUP

www.elevatedachievement.com