Standards for Mathematical Practice

The Practices in Action

Algebra I
Geometry
Algebra II
Integrated Math I
Integrated Math II
Integrated Math III

elevated achievement GROUP

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Introduction

When the Common Core State Standards for Mathematics were adopted in 2010, their stated purpose was to improve the level of mathematics achievement in the United States, which had once led the world but had fallen far behind that of other nations. This lag promised severe economic and technological consequences if American students weren't brought up to speed, and soon.

The Math Wars

> **"The mistakes and unresolved difficulties of the past in mathematics have always been the opportunities of its future."**
>
> —Eric Temple Bell

The new standards grew out of a long and heated debate about mathematics learning, whose pendulum had swung back and forth since the first American mathematics textbook was published in 1788. It raged through the "new math" movement of the 1950s and 60s, and continues to this day.

> "Within the first half century of the founding of the United States, the great school mathematics debate was established. Should teachers offer students rules and facts to memorize? Or should they give students material to reason about in order to discover and develop understanding of underlying mathematical principles?" (Larson & Kanold, 2016)

This ongoing debate became known as the "math wars," pitting conceptual understanding and sense-making against procedures, rules, and memorization. The new math standards grew out of decades-long attempts to acknowledge that both were important aspects of the math curriculum. By incorporating both Standards of Mathematical Content *and* Standards of Mathematical Practice, they brought together both sides of the math wars, building on "the best of previous state standards plus a large body of evidence from international comparisons and domestic reports and recommendations to define a sturdy staircase to college and career readiness." (National Governors Association, 2013)

Focus, Coherence, and Rigor

The standards also addressed the problem of a math curriculum that, over time, had become "a mile wide and an inch deep." To remedy this, the standards brought in three important elements: focus, coherence, and rigor.

Focus—narrowing the scope of content in each grade to a smaller set of clear and specific topics so that more time is devoted to each topic to promote deeper understanding and higher achievement

Coherence—the careful, deliberate, and progressive development of ideas based on how students' mathematical knowledge, skill, and understanding develop over time

Rigor—equally pursuing three important aspects of mathematical knowledge:

- Conceptual understanding (indicated by the term *understand*)
- Procedural skill and fluency (indicated by the term *fluently*)
- Applications (indicated by the term *real-world problems*)

For the content standards, this effort was a great success, resulting in a smaller number of standards per grade, research-based learning progressions, and a balance of concepts, procedures, and applications.

Unfortunately, the same effort was not applied to the practice standards, resulting in issues when putting them into practice. Although the practice standards have been narrowed to a manageable number and are the same for all grades, the topics are not clear or specific enough to provide concrete guidance on how they develop over time or how to support them from grade to grade. In summary, due to their lack of focus, coherence, and rigor, the practice standards remain unclear and difficult to implement successfully.

What the Standards Provide

The math standards brought together a vast array of research and instructional expertise to present a clear framework for mathematics instruction in the United States. Much of what they provide is a vast improvement over the skewed or overly complicated math standards of the past. But the standards cannot—and were never intended to—do it all.

> **"The essence of mathematics is not to make simple things complicated, but to make complicated things simple."**
>
> —Stan Gudder

THE STANDARDS DO...	THE STANDARDS DO NOT...
• Set grade-specific standards, providing clear signposts along the way to the goal of college and career readiness for all students. • Specify the knowledge and skills to be taught in each grade, based on state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. • Lay out a focused, coherent, and rigorous framework for content *learning*, which balances both sides of the "math wars."	• Define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations or the supports needed for English learners or students with special needs. • Dictate the specific way that mathematical content should be taught or the order of concepts within a grade level. • Provide the same level of focus, coherence, or rigor for mathematical practices, which are meant to be equally important.

Growing out of the math wars, the standards place a strong emphasis on the equal importance of procedural skill and mathematical understanding. And, they are clear in what they mean by mathematical understanding.

> "One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$." (National Governors Association, 2013)

Knowing this, it is easy to see why the new standards incorporated standards of two very different types.

Two Types of Standards

By articulating both content and practice, the standards define both what students should know and be able to do in mathematics and how they should be thinking about mathematics.

STANDARDS FOR MATHEMATICAL CONTENT	STANDARDS FOR MATHEMATICAL PRACTICE
A list of things students should understand and be able to do by the end of each grade • Specific mathematical knowledge and skills that follow a step-by-step learning progression across grade levels and courses • K-8 organized by grade level; high school organized by conceptual theme • Familiar to most teachers • Easily and frequently tested, and therefore the focus of the typical math curriculum	**A list of ways that proficient students engage with mathematics, including thinking skills and habits of mind** • More general processes and proficiencies that evolve over time, influenced by cognitive development and the sophistication of the content • Standards are the same across all grade levels • Not as familiar to teachers • Not as easily or frequently tested, and therefore often neglected in the math curriculum
Example (Grade 5) **Operations and Algebraic Thinking (5.OA)** Write and interpret numerical expressions. 1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	**Examples (All Grades)** 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.

While the authors of the standards state that the two types of standards are meant to be used together in an integrated way (particularly those that focus on conceptual understanding), they offer no clear path for doing so. They require students to apply a variety of metacognitive skills without fully defining what those skills are or how they can be supported. It's no wonder, then, that the practice standards are so often neglected in the math curriculum.

That's why this book was designed with an intentional focus on the Standards for Mathematical Practice, attempting to bring to them the same level of focus, coherence, and rigor that are found in the content standards and to show how they can be successfully integrated with content learning to elevate mathematical achievement.

Metacognition

Metacognition is a word that gets thrown around a lot in education research, where it is often touted as a powerful key to deeper and more meaningful learning. In practice, however, the concept is often vague and less than useful. "Thinking about thinking" is not exactly a helpful strategy to put in practice in the classroom.

But metacognition can't be dismissed as just a trendy buzzword. Recent research has shown that students who were taught metacognitive strategies made an average of eight months more progress than students who were not. And that was over the course of just one year. (Emeny, 2013) It's clear from this data that metacognition is important, but what is it really, and how can it be taught?

Metacognition and Student Ownership

Metacognition requires students to examine, externalize, and apply their thinking, "such as:

- What it means to learn something,
- Awareness of one's strengths and weaknesses with specific skills or in a given learning context,
- Planning what's required to accomplish a specific learning goal or activity,
- Identifying and correcting errors, and
- Preparing ahead for learning processes." (Chick, 2017)

Metacognition is related to the concept of student ownership—a mindset that leads to elevated academic achievement and that teachers can actively develop in themselves and in their students. Students who own their learning are not thinking on a superficial level. They can state what they are learning and why, can explain how they learn best, can articulate when they are learning and when they are struggling, and understand their role in any academic setting. This is one type of "thinking about thinking" that leads to greater academic success. (Crowe & Kennedy, 2018)

Supporting Metacognition in the Math Classroom

Support for metacognition in the math classroom often looks like either a lot of teacher-led modeling and thinking out loud or giving students loads of problems to solve in the hope that they will somehow discover the most useful approaches and strategies on their own.

Most teachers will tell you that in practice, neither of these approaches works very well. And they *really* don't work for students who are already struggling. For many students, it's better to "show them the [metacognitive] toolkit and teach them how to use it one tool at a time…teaching one's brain to control the thought processes it has for the purpose of directing it towards the management of their own learning." (Emeny, 2013)

Fostering metacognition requires a balance of explicit instruction, teacher modeling, student-centered exploration, and responsive coaching that helps students first learn the kinds of questions and thought processes they can apply, and then grow to use them on their own. These metacognitive skills come naturally to some students but not to others. Teachers must play an active role in teaching them and helping students *own* their mathematical learning.

Metacognition and the Strands of Math Proficiency

For too long, judgments about what constitutes "good" math instruction have been based on false dichotomies. Learning is either student centered or teacher directed, and its goal either conceptual understanding or procedural fluency. Both of these either/or propositions vastly oversimplify the complexity of what our brains need to know and to do to engage in mathematical thought. (Larson & Kanold, 2016)

Indeed, the influential report of the National Mathematics Advisory Panel found that "all-encompassing recommendations that instruction should be entirely 'student centered' or 'teacher directed' are not supported by research," and that "the curriculum must simultaneously develop conceptual understanding, computational fluency, and problem solving skills." (National Mathematics Advisory Panel, 2008)

That report underscored the key findings of another seminal report, *Adding It Up: Helping Children Learn Mathematics,* which found that, much like literacy, attaining proficiency in math requires learners to weave together multiple strands all at once. These key strands of mathematical proficiency drew on and further refined mathematical process standards that had been developed over the previous two decades by the National Council of Teachers of Mathematics. (National Research Council, 2001; National Council of Teachers of Mathematics, 2000)

Defining the Practices for Metacognition

The Standards for Mathematical Practice grew out of the NCTM process standards as well as the NRC strands of proficiency. They acknowledge that weaving together all of the strands of mathematical proficiency is a complex process that requires students to acquire both content knowledge as well as a variety of skills and practices.

NCTM Process Standards (2000)

Problem Solving— developing a variety of strategies to solve problems that arise in mathematics and in other contexts

Reasoning and Proof—using various types of reasoning to investigate, develop, and evaluate mathematical arguments and proofs

Communication—using the language of mathematics to communicate thinking and to evaluate the thinking of others

Connections—understanding how mathematical ideas interconnect and apply to other contexts

Representation—using mathematical representations to solve problems and to model, interpret, and communicate ideas

(National Council of Teachers of Mathematics, 2000)

Standards for Mathematical Practice (2010)

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

(National Governor's Association, 2010)

National Research Council Strands of Proficiency (2001)

Conceptual Understanding—comprehension of mathematical concepts, operations, and relations

Procedural Fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

Strategic Competence—ability to formulate, represent, and solve mathematical problems

Adaptive Reasoning—capacity for logical thought, reflection, explanation, and justification

Productive Disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

(National Research Council, 2001)

As teachers, helping students do this cognitive work is just as complex. It requires us to understand the processes involved—to think about the thinking—and thereby help students develop the necessary conceptual understandings as well as the procedural and metacognitive skills required to be successful with math. The Standards for Mathematical Practice present a helpful framework for this complex and important work.

The practice standards are the same for all grade levels and represent ways that students can engage with mathematics. A few examples are included of how students at different levels or studying different topics might apply or demonstrate the proficiencies described. But while these brief descriptions are helpful for figuring out what the standards mean, they are not specific enough to provide much practical guidance for classroom implementation. In short, the practice standards lack the elements of focus, coherence, and rigor that the standards as a whole were created to achieve. In order to bring clarity around each element as they relate to the practice standards, let's examine the goals and issues for each.

GOALS	ISSUES
Focus—narrowing the scope of content in each grade to a smaller set of clear and specific topics so that more time is devoted to each topic to promote deeper understanding and higher achievement	The practice standards have been narrowed to a manageable number, but the topics are not clear or specific enough to provide concrete guidance.
Coherence—the careful, deliberate, and progressive development of ideas based on how students' mathematical knowledge, skill, and understanding develop over time	The practice standards are the same for all grades. Other than a few examples of what younger and older students do, there is no indication of how the proficiencies develop over time or how to support them from grade to grade.
Rigor—equally pursuing three important aspects of mathematical knowledge: 1. Conceptual understanding (indicated by the term *understand*) 2. Procedural skill and fluency (indicated by the term *fluently*) 3. Applications (indicated by the term *real-world problems*)	While the content standards attempt to balance all three aspects of mathematical knowledge, the practice standards are ways in which students could engage with all three. They require students to apply a variety of metacognitive skills without fully defining what those skills are or how they can be supported. The authors state that the practice standards need to connect with the content standards (particularly those that focus on conceptual understanding) but offer no clear path for doing so.

The Standards for Mathematical Practice

The Standards for Mathematical Practice "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students." For each one, a short description is offered, which states what mathematically proficient students do and how they demonstrate the "practices and proficiencies" outlined by the standards. (National Governor's Association, 2010)

THE STANDARDS FOR MATHEMATICAL PRACTICE	
Practice 1	Make sense of problems and persevere in solving them.
Practice 2	Reason abstractly and quantitatively.
Practice 3	Construct viable arguments and critique the reasoning of others.
Practice 4	Model with mathematics.
Practice 5	Use appropriate tools strategically.
Practice 6	Attend to precision.
Practice 7	Look for and make use of structure.
Practice 8	Look for and express regularity in repeated reasoning.

Understanding Their Structure

The practice standards overlap, both with content standards and each other. Structuring the practice standards into categories can help reveal the relationships at work.

OVERARCHING HABITS OF MIND OF A PRODUCTIVE MATHEMATICAL THINKER		
1 Make sense of problems and persevere in solving them. 6 Attend to precision.		
Reasoning and Explaining	**Modeling and Using Tools**	**Seeing Structure and Generalizing**
2 Reason abstractly and quantitatively. 3 Construct viable arguments and critique the reasoning of others.	4 Model with mathematics. 5 Use appropriate tools strategically.	7 Look for and make use of structure. 8 Look for and express regularity in repeated reasoning.

It is also important to maintain the integrity of each practice standard as a whole. While it may seem natural to teach each part of the standard as distinct practices, these parts were intentionally designed to promote metacognition through integrated application of those practices. For example, Mathematical Practice Standard 1 says, "Make sense of problems and persevere in solving them." It does not say, "Solve problems." Or "Make sense of problems." Therefore it is critical to provide students with opportunities designed to build their perseverance in course-appropriate ways by solving problems that require them to persevere to a solution beyond the point when they would like to give up. (Achieve the Core, 2019)

Using this Book

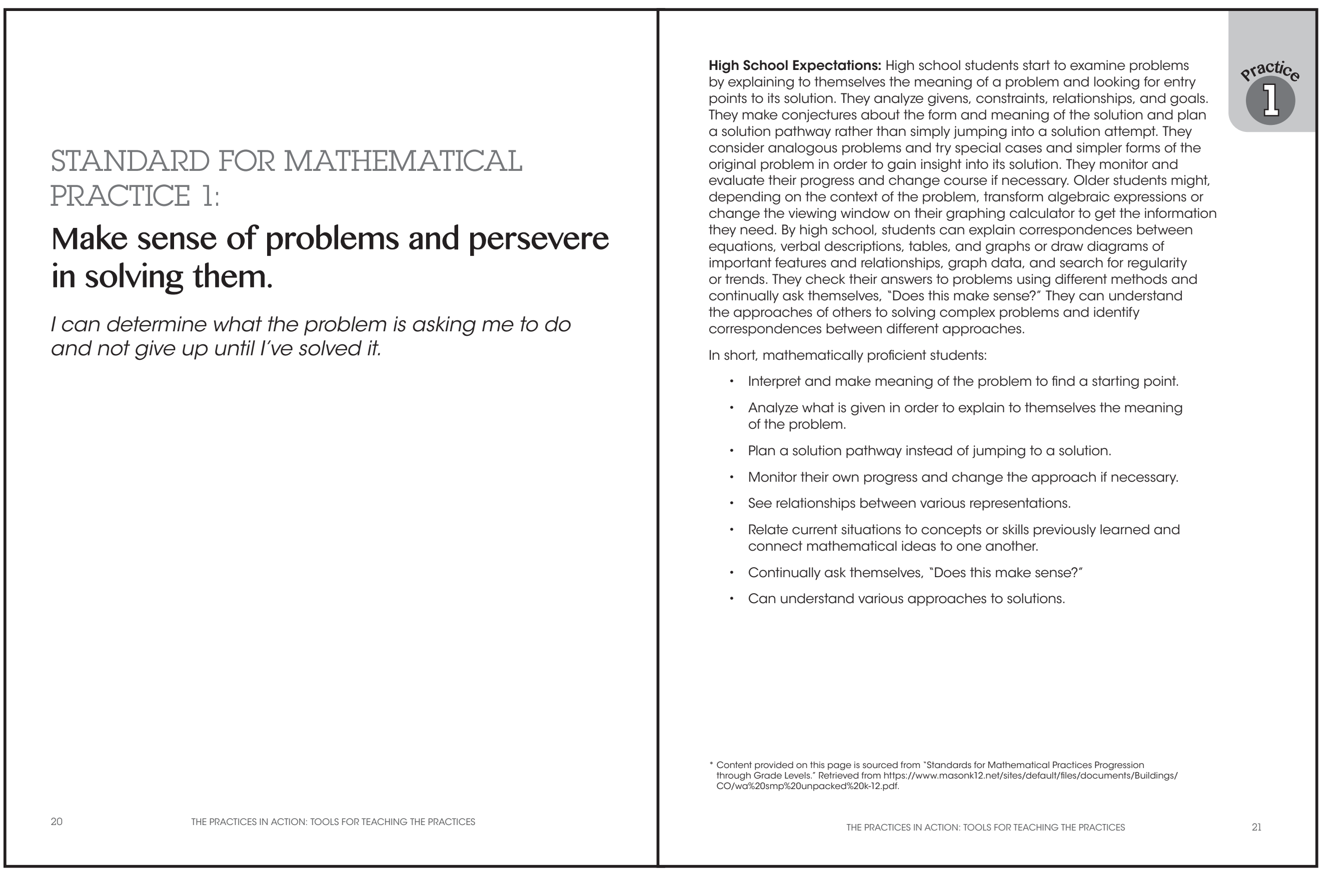
The rest of this book will present some other ways to bring focus, coherence, and rigor to the Standards for Mathematical Practice by providing more specific high school expectations, and concrete ways that teachers can support the cognitive and metacognitive skills students need to weave together the mathematical knowledge, skills, and practices that lead to true proficiency.

The book is organized into two sections. The first provides tools for teaching the practices and is organized by each Standard for Mathematical Practice. The second provides examples of what the practices look like when they are put into action and is organized by each high school math course.

Tools for Teaching the Practices

The pages in this first section include the following types of tools for each standard:

1. The **Standard for Mathematical Practice** along with specific **High School Expectations.**



STANDARD FOR MATHEMATICAL PRACTICE 1:

Make sense of problems and persevere in solving them.

I can determine what the problem is asking me to do and not give up until I've solved it.

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Practice 1

High School Expectations: High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

In short, mathematically proficient students:

- Interpret and make meaning of the problem to find a starting point.
- Analyze what is given in order to explain to themselves the meaning of the problem.
- Plan a solution pathway instead of jumping to a solution.
- Monitor their own progress and change the approach if necessary.
- See relationships between various representations.
- Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another.
- Continually ask themselves, "Does this make sense?"
- Can understand various approaches to solutions.

* Content provided on this page is sourced from "Standards for Mathematical Practices Progression through Grade Levels." Retrieved from https://www.masonk12.net/sites/default/files/documents/Buildings/CO/wa%20smp%20unpacked%20k-12.pdf.

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2. A step-by-step **Process** to teach each standard that teachers can use for initial instruction, modeling, and guiding students to master the practices. And, a **Reflection Guide** for each standard to support students as they "think about their thinking" and where they are in the mastery of the practice.

Practice 1

Make sense of problems and persevere in solving them.

"I can determine what the problem is asking me to do and not give up until I've solved it."

Process

Process to make sense of problems

1. Read the problem out loud.
2. Identify and clarify each word that tells you what to do mathematically.
3. Explain the problem in your own words.
4. Explain how you will know you have solved the problem correctly.

Process to persevere in solving them

1. Make a plan for solving the problem.
2. Begin to solve the problem.
3. Each time you get stuck, identify where you got stuck.
4. Ask for help, as needed.
5. Keep working until you've solved the problem correctly.

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Practice 1

Make sense of problems and persevere in solving them.

"I can determine what the problem is asking me to do and not give up until I've solved it."

Reflection

To what degree can you determine what the problem is asking you to do and not give up until you've solved it?

1	2	3	4	5
never		sometimes		always

- What does "make sense of problems" mean?
- What does "persevere in solving them" mean?
- How do you determine what the problem is asking you to do?
- How do you not give up until you've solved the problem?
- How does "making sense of problems and persevering in solving them" help you?

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Course-Level Examples of the Practices in Action

While the high school specific expectations lend considerably more clarity and specificity than are found in the Standards for Mathematical Practice, they can still be a little abstract. They are expanded and made more concrete in this section with the following three features:

- **The Practice in Action**—what proficiency looks like at the course level and how it connects with the content standards
- **Questions to Foster Metacognition**—prompts teachers can use to help students develop their metacognitive skills
- **Ownership Statements**—what students will say to show they are applying the practice standard and developing ownership of their learning

This expanded support not only gives a more detailed view of what the expectations actually look and sound like in the classroom, but it also shows how each practice standard relates to the specific math content students are learning. The examples that follow are for the same practice standard, Make sense of problems and persevere in solving them.

Practice 1

Make sense of problems and persevere in solving them.

Algebra I

The Practice in Action: When presented with a problem relating to slope, Algebra I students utilizing this practice explain the meaning of the problem and look for entry points to its solution. They also check their answers by graphing and asking themselves "Does this make sense?"

Find the slope of a line that goes through ordered pairs (4, 2) and (–3, 16).

TEACHER: What is the problem asking you to do?

STUDENT: *The problem is asking me to find the slope of a line using two sets of points.*

TEACHER: What information are you given in the problem?

STUDENT: *I was given the two ordered pairs which are four, two and negative three, sixteen.*

TEACHER: What is your plan to solve the problem?

STUDENT: *I start by reminding myself that in an ordered pair, the first number represents* x *and the second represents* y*. I also remember that slope is defined as rise over run and that rise over run is change in* y *divided by change in* x *which is also called* m*. This forms the formula for slope. I substitute the two sets of points given in the problem into that formula and I can solve the problem.*

TEACHER: What other strategies might you try?

STUDENT: *I can solve it another way by graphing, which I've done before. Understanding various approaches to solutions is important because it helps me make better sense of what I'm doing and helps me to persevere if I struggle.*

TEACHER: How does making sense of problems and persevering in solving them help you?

STUDENT: *Making sense of problems and persevering in solving them helps me understand what the problem is really asking me to do. Then, I can figure out how to solve it and work on it until it was finished.*

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Questions to Foster Metacognition:

- What is the problem asking you to do?
- What is your plan to solve the problem?
- How would you explain what the problem is asking you to do in your own words?
- What information is given in the problem?
- What steps in the process are you most confident about?
- What are some other strategies you might try?
- Why is being able to interpret and make meaning of the problem important?
- How does analyzing the information given help you?
- Why is planning a solution pathway instead of jumping to a solution important?
- Why is monitoring your progress and changing the approach, if needed, important?
- Why is being able to see relationships between various representations important?
- Why is understanding various approaches to solutions important?
- Why is continually asking yourself, "Does this make sense?" important?
- How does making sense of problems and persevering in solving them help you?

Ownership Statements:

- Being able to interpret and make meaning of the problem is important because ________.
- Being able to look for starting points is important because ________.
- Analyzing what information is given helps me because ________.
- Planning a solution pathway instead of jumping to a solution is important because ________.
- Monitoring my progress and changing the approach, if needed, is important because ________.
- Being able to see relationships between various representations is important because ________.
- Understanding various approaches to solutions is important because ________.
- Continually asking myself, "Does this make sense?" is important because ________.
- Making sense of problems and persevering in solving them helps me ________.

THE PRACTICES IN ACTION: ALGEBRA I 55

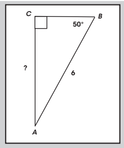
Practice 1

Make sense of problems and persevere in solving them.

Geometry

The Practice in Action: When presented with solving for a side in right triangles, Geometry students utilizing this practice explain the meaning of the problem, look for entry points to its solution and find the process for finding a solution. They also check their answers by asking themselves "Does this make sense?"

Determine which trigonometric ratio to use when solving for a side in the right triangle provided.



TEACHER: What is the problem asking you to do?

STUDENT: *The problem is asking me to determine which trigonometric ratio I need to use to solve for a side in the triangle.*

TEACHER: What information is given in the problem?

STUDENT: *The problem gives the measurement of one angle and the length of the hypotenuse. Since the problem states that it is a right triangle, I know that another angle is 90 degrees.*

TEACHER: What is your plan to solve the problem?

STUDENT: *I start focusing on Angle* B *since that is the angle that is explicitly given in the diagram. I notice that I am given the length of the hypotenuse, and am asked to find the length of the side opposite Angle* B*. The trigonometric ratio that contains both of those sides is the sine since the sine ratio is the ratio of the length of the opposite side divided by the length of the hypotenuse.*

TEACHER: How does making sense of problems and persevering in solving them help you?

STUDENT: *Making sense of the problem and persevering in solving it helpsme to understand what the problem is really asking me to do. Then, I can figure out how to solve it and then work on it until it was finished.*

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Questions to Foster Metacognition:

What is the problem asking you to do?
What is your plan to solve the problem?
How would you explain what the problem is asking you to do in your own words?
What information is given in the problem?
What steps in the process are you most confident about?
What are some other strategies you might try?
Why is being able to interpret and make meaning of the problem important?
How does analyzing the information given help you?
Why is planning a solution pathway instead of jumping to a solution important?
Why is monitoring your progress and changing the approach, if needed, important?
Why is being able to see relationships between various representations important?
Why is understanding various approaches to solutions important?
Why is continually asking yourself, "Does this make sense?" important?
How does making sense of problems and persevering in solving them help you?

Ownership Statements:

Being able to interpret and make meaning of the problem is important because ________.
Being able to look for starting points is important because ________.
Analyzing what information is given helps me because ________.
Planning a solution pathway instead of jumping to a solution is important because ________.
Monitoring my progress and changing the approach, if needed, is important because ________.
Being able to see relationships between various representations is important because ________.
Understanding various approaches to solutions is important because ________.
Continually asking myself, "Does this make sense?" is important because ________.
Making sense of problems and persevering in solving them helps me ________.

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As these examples show, students are learning the language of mathematics and the course-level math content at the same time they are learning the metacognitive skills of examining and applying their own reasoning.

> "For students to become more metacognitive, they must be taught the concept and its language explicitly.... Metacognition is not generic, but instead is most effective when it is adapted to reflect the specific learning contexts of a specific topic, course, or discipline." (Chick, 2017)

Each example demonstrates how these cognitive and metacognitive skills are interconnected and interdependent—they can and should be taught together.

Questions to Foster Metacognition

It would be a rare classroom indeed in which students spontaneously started using Ownership Statements. On the contrary—most students need a great deal of explicit instruction, modeling, and coaching before they develop the kinds of metacognitive skills that allow them to take ownership of their learning.

The **Questions to Foster Metacognition** in each grade-level section are designed to help students along that path. These can be thought of as "what teachers should be saying"—or, more accurately, ideas teachers can use to help students apply the practice standards in grade-appropriate ways.
For example:

- What is the problem asking you to do?
- What is your plan to solve it?
- What tools could you use to help you?
- What are some other ways to solve it?

Questions like these help students expand their thinking, develop metacognitive skills, and become more aware of their own strengths and weaknesses. They are one of the "explicit and concerted ways we make students aware of themselves as learners. We must regularly ask, not only 'What are you learning?' but 'How are you learning?' We must confront them with the effectiveness (more often ineffectiveness) of their approaches. We must offer alternatives and then challenge students to test the efficacy of those approaches." (Weimer, 2012)

Later, these questions can and should be handed over to the students themselves. For example, the teacher can model asking a question and then encourage students to ask each other the same question, followed by a student self-reflection. All that is required is a quick change in pronoun. For example:

- The teacher asks a student, "What is the problem asking you to do?"
- A student asks his or her partner, "What is the problem asking you to do?"
- A student asks himself or herself, "What is the problem asking *me* to do?"

In this book, we provide the initial question but recognize the need for a teacher to model how to use the same question to drive cooperative and collaborative group work and self-reflection. It is ultimately when students begin using these kinds of questions to monitor their own thinking processes that they will own their learning and be well on their way to math proficiency.

> **"Mathematics is not about numbers, equations, computations, or algorithms: it is about understanding."**
>
> —William Paul Thurston

Ownership Statements

Imagine walking into a first-grade classroom and asking a student, "What are you learning?" The student answers, "I am making a picture of a number." It's an acceptable answer but one that does not convey much about the context, content, or skills associated with the learning. This is a student who is simply "doing school," completing the task at hand but not really understanding why.

You move on to another student and ask the same question. This student answers, "I am working with my elbow partner. We are making models of our addition problems. Then we write them using numbers." That's better, and indicates a student who has progressed to "understanding school," one who understands and can explain the skill or strategy on a surface level.

Now imagine asking a third student and hearing this: "We're learning that the numbers on both sides of the equal sign must be the same. You can't use an equal sign if this is not true. Today we have to tell each other if the problem is true or false. Then we make a model with blocks to show if it is true. If it is true, our model shows this. If it is false, we have to fix it. This model helps us confirm or correct our solution."

Wow. Now imagine that every student in that class could answer at the same depth. That is what it sounds like when students are taking ownership of their learning. (Crowe & Kennedy, 2018) They not only understand the content, but are actively integrating:

- Both cognitive and metacognitive skills.
- All strands of mathematical proficiency.
- Mathematical content with mathematical practice.

The **Ownership Statements** on the second page of each grade-level section are a guide to the kinds of talk you will hear in a classroom in which students are actively using the mathematical practice. You can think of this section as "what students should be saying." Some examples are:

- I can solve the problem by _________.
- I can explain how I got the answer.
- When I am stuck, I can _________.

These are the kinds of statements that demonstrate students are utilizing the mathematical practices and developing ownership of their learning.

Students who are developing ownership of the mathematical practices…

- State what the problem is asking.
- Explain their plan to solve the problem.
- Share how they solved the problem.
- Ask questions about their strategies and solutions.
- Answer questions.
- Clarify that they know that there may be more than one answer.
- Acknowledge if an answer makes sense.
- Determine what they need to do to get the answer.

Tools for Teaching the Practices

The Standards

Practice 1	Make sense of problems and persevere in solving them.
Practice 2	Reason abstractly and quantitatively.
Practice 3	Construct viable arguments and critique the reasoning of others.
Practice 4	Model with mathematics.
Practice 5	Use appropriate tools strategically.
Practice 6	Attend to precision.
Practice 7	Look for and make use of structure.
Practice 8	Look for and express regularity in repeated reasoning.

Student Ownership Statements

Practice 1	*I can determine what the problem is asking me to do and not give up until I've solved it.*
Practice 2	*I can make sense of quantities and use math symbols, numbers, or words to represent and solve problems.*
Practice 3	*I can justify my conclusions with evidence from my work, and I can listen to or read others' arguments and decide if they make sense.*
Practice 4	*I can use what I know about math symbols, words, pictures, tools, and diagrams to solve everyday problems.*
Practice 5	*I can determine which tools are the right ones to use when solving problems.*
Practice 6	*I can communicate precisely what I'm doing and explain my thinking using mathematical language.*
Practice 7	*I can determine overall structures and patterns to help me solve problems.*
Practice 8	*I can use what I already know about problem solving strategies, patterns, and other shortcuts to solve problems.*